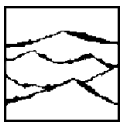

WAVECREST Corporation

A NEW METHOD FOR JITTER DECOMPOSITION THROUGH DISTRIBUTION TAIL-FITTING

Technical Bulletin No. 9

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A New Method for Jitter Decomposition Through Distribution Tail Fitting

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In this paper, *WAVECREST Corporation* presents a new time-domain jitter separation method that automatically searches and fits the tail parts of the jitter histogram with nonlinear jitter models and also estimates deterministic and random jitter components. Bit error rate (BER) calculation, based on the deterministic and random jitter components, is discussed and demonstrated.

This method/algorithm is used to decompose a total jitter histogram into DJ and RJ components through tail fitting. Also discussed is total jitter histogram theory and its relationship with DJ and RJ processes. Also described is an algorithm that automatically identifies the tail parts of the distribution and fits them with the Gaussian distributions for estimating the DJ and RJ components. *WAVECREST*'s Monte Carlo simulation results that verify the validity of our algorithm will be presented. Practical application examples are given using *WAVECREST*'s jitter analysis software which incorporates the algorithm and verifies the correlation as well as demonstrates the advantage of having DJ and RJ components.

Note: Theoretical analysis and computer simulations have been performed. The tail search and fitting algorithms have been implemented in our current jitter analysis software product, *Virtual Instruments Signal Integrity*™ (VISI). A patent application has been filed with the U. S. Patent Office.

Introduction

In high-speed signal generation, transmission and reception, jitter is generally defined as any timing displacement (or error) referenced to the timing of an ideal signal. There are many ways to classify various jitters with many components making up each classification. The most commonly used are statistical-based or signal property-based (e.g., pattern dependent). In the statistical-based classification, jitters are split into two categories: deterministic jitter (DJ) and random jitter (RJ). Based on their definitions, DJ is bounded while RJ is unbounded. In the signal property-based classification, jitters are split into noise jitter (random motion of electrons) (NJ) and pattern dependent jitter (PDJ). These classification schemes are broad and are the first step in identifying jitter. For a general review of jitter and related issues, please see references [1][2][3] in the References section. A hybrid approach to jitter classification will be used which covers most jitter sources.

As previously mentioned, jitter can be classified within each broad category. Deterministic jitter can be further separated into duty cycle distortion (DCD), inter-symbol interference (ISI), periodic (PJ) and bounded uncorrelated (BU) components. Jitter can be a single Gaussian (SG) or multiple Gaussian (MG) within the random jitter. Each jitter component has specific corresponding root causes and characteristics. General jitter sources for DJ are: lossy media,

reflections, cross talk, electromagnetic interference (EMI), systematical modulations and pattern dependency. General jitter sources for RJ are: thermal noise, shot noise, flick noise, random modulation and non-stationary interference. In real world applications, generally speaking, what is measured is the total jitter; both DJ and RJ mixed together. To understand root causes of jitter, separating and identifying each jitter component is essential. As the speed for microprocessor, memory, data bus and transmission media steadily increases, failures caused by jitter will become more and more severe while modeling, simulating, measuring and characterizing high-speed signal jitter will become even more important and challenging.

WAVECREST Corporation has developed a method for DJ and RJ separation based on the Blackman-Tukey algorithm [4]. With this method, the DCD+ISI is obtained by calculating the mean of the time error (measured edges versus that expected from a pattern). PJ and RJ are calculated through Fast Fourier Transform (FFT) spectrum estimations of the variance using the auto-correlation function of the time jitter. Since spectrum estimation is required in this method, a time series record of jitter measurements is needed. In the case when a jitter time record (i.e., jitter is measured as a function of time) is unavailable, this method may not apply.

For time domain measurements, jitter can be measured for a specific edge transition or over a time span of many edge transitions. In each case, many jitter samples are collected for each edge transition so that statistical information can be gathered and analyzed. A typical example in real world practice is the jitter histogram measured for a specific edge transition or jitter histograms measured over a time span of many edge transitions. Such a jitter histogram reflects the mixture of DJ and RJ processes associated with the edge transitions. For many years, this information has been available, but no theory or method had been was established to decompose the total jitter histogram into DJ and RJ components. What has been used to quantify jitter is the statistical peak-to-peak value and 1σ standard deviation based on the entire histogram distribution that has both DJ and RJ components. The correct way to quantify jitter is to use the peak-to-peak value for DJ, since it is bounded, and to use 1σ standard deviation for RJ, since it is unbounded and random. It can be seen that the use of the peak-to-peak value and 1σ standard deviation based on a total histogram that has both DJ and RJ components is not only misleading but statistically wrong.

Jitter Histogram Distribution and the Relationship with DJ and RJ

Physically, random jitter is due to the random motion of particles within a device or transmission media. Theoretically, the tail part of the histogram distribution reflects the random jitter process. A Gaussian distribution best describes the random velocity of these particles in an equilibrium state. Therefore, random jitter is naturally modeled by a Gaussian function. Since multi-temperature particle distribution is possible, a multi-Gaussian distribution function is needed to model certain random jitter processes.

A single Gaussian jitter histogram distribution is defined as

$$h(t) = N_{\max} e^{\frac{-(t-\mu)^2}{2\sigma^2}} \qquad (1),$$

where N_{max} is maximum event count, t is the jitter, μ and σ are the Gaussian mean and standard deviation respectively.

The measured, total jitter histogram represents the scaled-up, total jitter probability distribution function (PDF) and, if RJ and DJ processes are independent, the convolution of RJ PDF with DJ PDF gives the total PDF. In most cases, such an assumption is valid, therefore, the tail part of the distribution should be mainly determined by the random jitter, which, in general, has a Gaussian-type distribution. The random noise can be quantified by the 1σ standard deviation of Gaussian distribution while the DJ can be quantified by the peak-to-peak value. Depending on the error probability level, the total RJ can be a multiple of the σ, deduced from the Gaussian distribution.

In the absence of DJ, the histogram of jitter should be, roughly, a Gaussian distribution. Under this condition, there is only one peak in the distribution which corresponds to zero DJ. The rms RJ is the σ value. When DJ and RJ both come into play, the measured jitter distribution will be broadened and no longer Gaussian as a whole. On the other hand, both ends of the distribution should still retain Gaussian-type tails since DJ PDF is bounded. These tail-part distributions can be used to deduce the RJ number. Because of the DJ, the mean of each tail changes and multi-peaks can be present in the histogram. The jitter difference between the far left peak value and far right peak value is the DJ. Figure 1 is a drawing of such a broadened total histogram in the presence of both DJ and RJ.

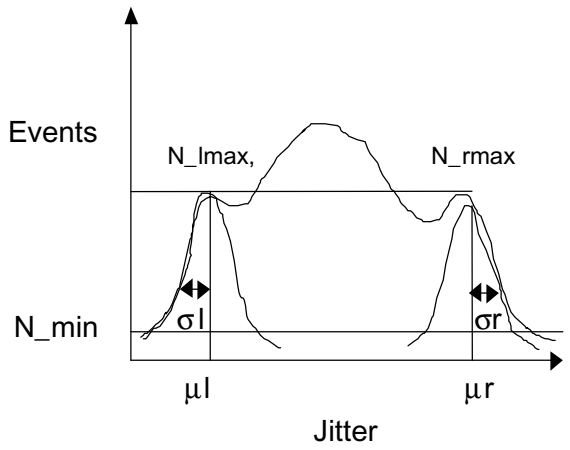

Figure 1 - Drawing of total jitter histogram in the presence of DJ and RJ.

If there is no bias and statistical sampling noise in the measurement, the two tails, which represent the random process, should be symmetrical. Since it is not possible to completely randomize measurements and reduce the sampling noise to zero, the σ values for the far left and far right Gaussian tails may not be the same. The total RJ value should be the average of these two tails while DJ is the distance between the two peaks of the far left and far right Gaussian tails, namely,

$$RJ = (\sigma_l + \sigma_r)/2 \qquad (2)$$

and

$$DJ = \mu_r - \mu_l \qquad (3).$$

Tail Search and Fitting

Requirements and Specifications

Identifying the tail parts of the histogram distribution and then to fit them with the Gaussian function are the key to DJ and RJ separation of a given total jitter histogram. It is not possible to tell where the tail part of the histogram will be without studying each individual datum and its relationship with the neighboring data. The easiest way to identify a tail part is through the graphical display of the histogram and visually identifying the tail part. The disadvantage of such an approach is that it lacks repeatability and cannot be adopted for production test. Therefore, the requirements for a search algorithm should be: i.) Capable of finding the true tail part quickly, accurately and repeatedly, ii.) Automatic (i.e., no user intervention or visual inspection are required).

The fitting procedure should be able to deal with statistical fluctuations while factoring this into the fitting routines. The tail part has the lowest event counts and statistical uncertainty can be high. Simple, straight forward, least-square fit algorithms will not work since the statistical error will propagate into the fitting parameters. This in turn gives rise large to errors in DJ and RJ estimation. A more advanced non-linear fitting algorithm is needed to meet these requirements.

Algorithm for Tail Identification

One of the key characteristics of Gaussian tail-fit is its monotonicity. That means: for the left side of the tail, it monotonically increases; for the right part of the tail, it monotonically decreases. However, due to the presence of DJ, monotonicity will break down producing local maximums near the left and right part of tails. Without DJ, there is only one maximum that corresponds to the mean of the distribution.

One challenge that a tail search algorithm faces is statistical fluctuations. In the presence of statistical fluctuations, the monotonicity of a real Gaussian

distribution is no longer true. Using the raw fluctuating data to find the local maximum points for both left and right tails is extremely difficult, if not impossible. The solution is to first filter out the noise and then use the smoothed histogram to locate the maximum points. In general, there are two ways to achieve this. One is through direct time domain averaging. Another is through FFT to get the spectrum, apply a low-pass filter and then apply the IFFT. In time domain averaging, deciding how many data points to use is important since this will determine the smooth level of the curve. Also, in the FFT/IFFT approach, the bandwidth of the filter has to be determined. The number of averaging points and filter bandwidth may need to be adjusted, depending on the fluctuation noise frequency and amplitude. In other words, a rule-based, artificial intelligent algorithm must be used to enable the smoothing algorithm to accommodate a wide range of fluctuation amplitudes and frequencies. This is an important requirement to guarantee that smoothing only washes away the unwanted fluctuation noise and not the true feature of the jitter histogram.

Once the smoothed histogram $hs(t)$ is obtained, either through the time domain averaging or time-frequency domain FFT-filtering-IFFT, the maximum locations can be found by calculating the first and second order derivatives of the jitter histogram. The only maximum points of interest are the first maximum from the far left and the first maximum from the far right.

Algorithm for the Tail Fitting

A fitting algorithm should be used which weights the data record based on the quality of each data. The larger the error, the smaller role it should play in minimizing the difference between the expected model value and the measured value. Thus, we need to use χ^2 as a gauge to determine how good the fit is. The fitting function is Gaussian and the fitting algorithm is nonlinear thereby handling both linear and non-linear fitting functions. Please see reference [5] for details of χ^2 theory.

In the case of linear least-squared fitting, in contrast to linear equation solving, χ^2 fitting is an iterating process. The final answer is obtained when the iteration converges. For this reason, initial values of the fitting parameters are needed. A primitive way to do this is to input different initial values and to see whether they converge to the same final values. If the initial guessed values are far from the final actual values, it may either take a longer time to converge or get stuck at a local χ^2 minimum and never converge to the final global χ^2 minimum point. Calculations should be carried out to estimate the initial fitting parameters using the tail parts of histogram so that the initial fitting parameters are close to the final converging values. This will also cause the iteration to converge rapidly and to avoid stuck-in local minimum (pivot).

MONTO CARLO SIMULATIONS

Histogram with Statistical Noise

Simulations, using a known bimodal histogram represented by two added Gaussian distributions superimposed with random noise, are needed to test how well the search and fitting algorithms worked. This causes the overall histogram to be close to that of actual measurements. The overall histogram is represented by the following equation:

$$hi(t)=N_1 e^{-\frac{(t-\mu_l)^2}{2\sigma_l^2}} + N_r e^{-\frac{(t-\mu_r)^2}{2\sigma_r^2}} + N_n ran(t) \quad (4)$$

where N_1, N_r are the peak values, μ_l, μ_r are means, and σ_l, σ_r are standard deviations for two Gaussian distributions. $ran(t)$ is a random number-generating function based on Monte Carlo methodology with a mean of zero and a standard deviation of unity. N_n is the amplitude for the random number envelopes. For Monte Carlo-based random number generation method, please see reference [6].

A good search and fitting algorithm should return the fitted parameters that are consistent with those pre-defined in the simulation. A critical test is whether an accurate fitting parameter can be obtained in the presence of significant statistical fluctuations, i.e., N_n is a significant portion of N_1 or N_r. Otherwise, no accurate parameters can be obtained since all the real world measurements are subject to statistical fluctuation.

Fitting Results

There are two Gaussian distribution scenarios that need to be addressed. The first is when two Gaussian distributions are well separated, i.e., when $\mu_r - \mu_l > \sigma_l + \sigma_r$. Under this condition, the two distributions are not well mixed and the tail parts, up to the point of the first maximum, are essentially uncontaminated. Therefore, both left and right tail data from the lower value to the first maximums can be used. This will enhance the tail data usage and the Gaussian model will be better constrained. This can correspond to the case when DJ > 2 RJ in the jitter analysis applications. Such a histogram is very common in spread-spectrum clock devices.

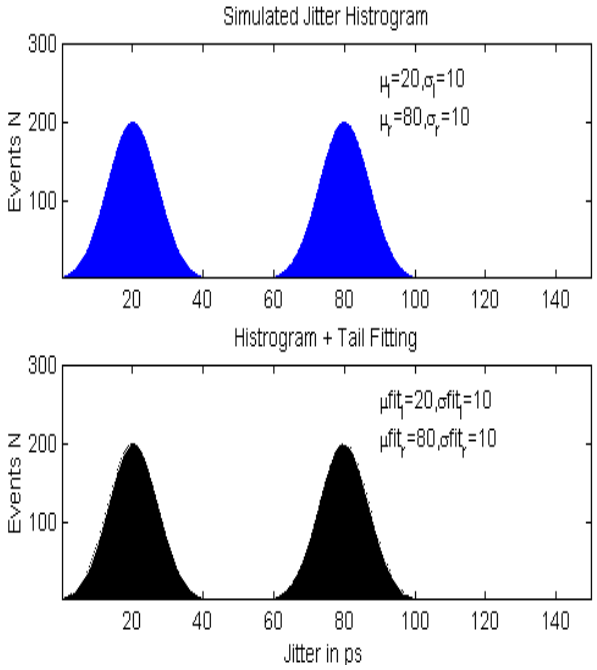

Figure 2.1 - Nn = 0, no statistical fluctuation.

The second scenario is when two Gaussian distributions are not well separated, namely $\mu_r - \mu_l < \sigma_l + \sigma_r$. Under this condition, the contamination of two distributions could extend to the tail parts. As a result, using only the lower parts of the tails for the fitting will minimize contamination. A conservative way is to use the tail part from the lowest event count to half of the N_l or N_r. This can correspond to the case when DJ < 2 RJ in the jitter analysis applications.

Figures 2.1-2.2 show the results corresponding to two well-separated Gaussian histograms (i.e., $\mu_r - \mu_l > \sigma_l + \sigma_r$). Figures 3.1-3.2 show the results corresponding to two mixed Gaussian histograms (i.e., $\mu_r - \mu_l < \sigma_l + \sigma_r$). In each case, non-fluctuated ($N_n = 0$) and fluctuated ($N_n \neq 0$) scenarios are considered. In both cases, the fitted parameters are consistent with the simulated parameters to within 4%, even when the statistical fluctuation reaches 15% of the total histogram peak.

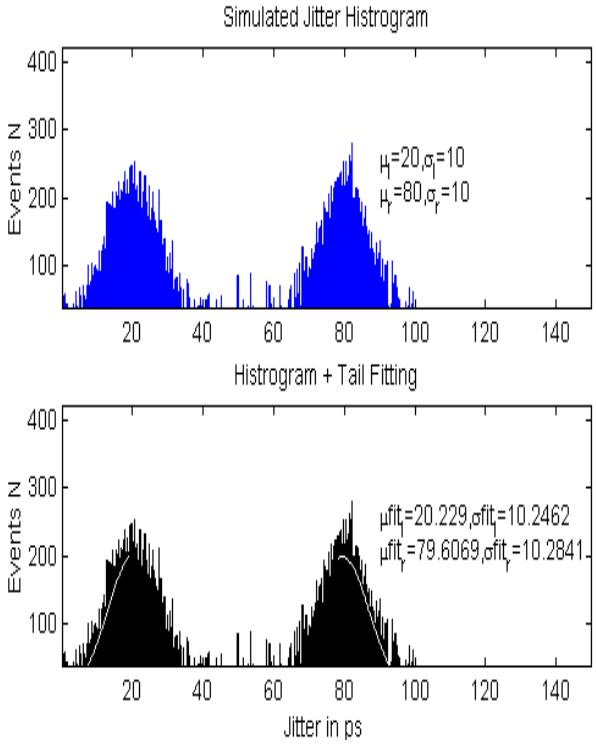

Figure 2.2 - $N_n = 30$, with significant statistical fluctuation

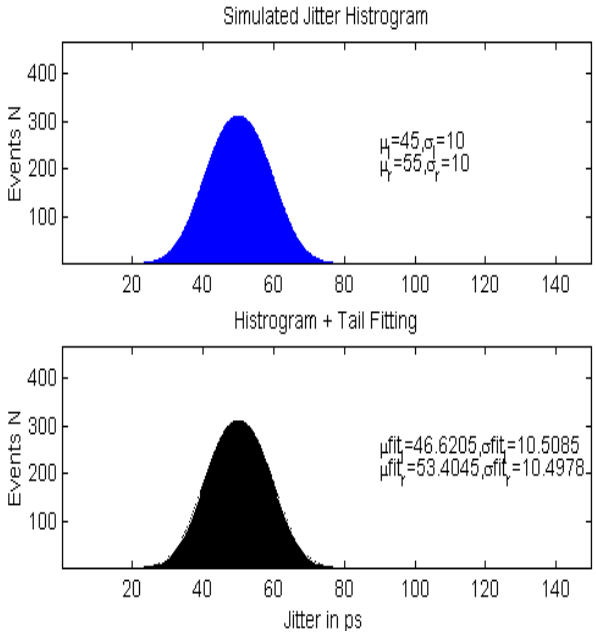

Figure 3.1 - $N_n = 0$, no statistical fluctuation.

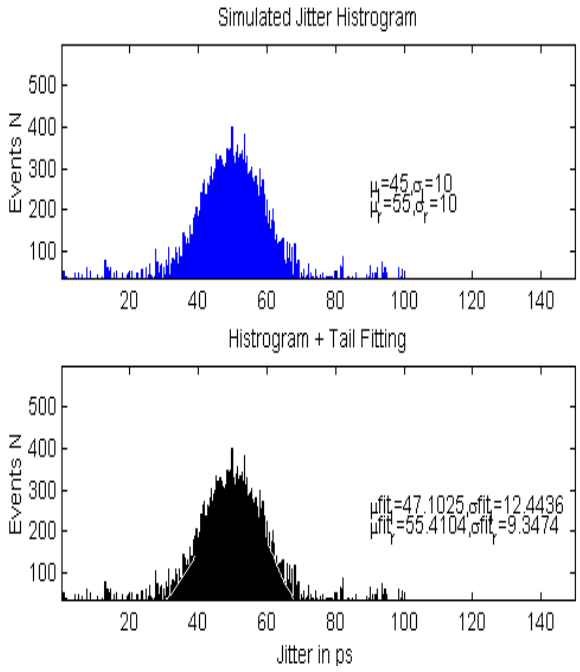

Figure 3.2 - $N_n = 30$, with significant statistical fluctuation.

A Practical Case Study

The search and fitting algorithms previously discussed have been implemented in *WAVECREST*'s Virtual Instrument Signal Integrity™ (VISI) software. Using the *WAVECREST* DTS-2075™ system and a clock input to the instrument, the jitter histogram was measured and VISI was used to decompose DJ and RJ. Figure 4 shows an example of clock jitter histogram and DJ and RJ values deduced from the tail-fitting algorithm. In this case, the histogram shows twin-peak DJ process caused by a periodic modulation: A 100MHz clock signal with a 5MHz periodic modulation. Thick lines indicate the Gaussian model fitted to the tail part of the distribution and overlaying the measured histogram (thin lines). The `DJpk-pk` value represents the modulation jitter amplitude.

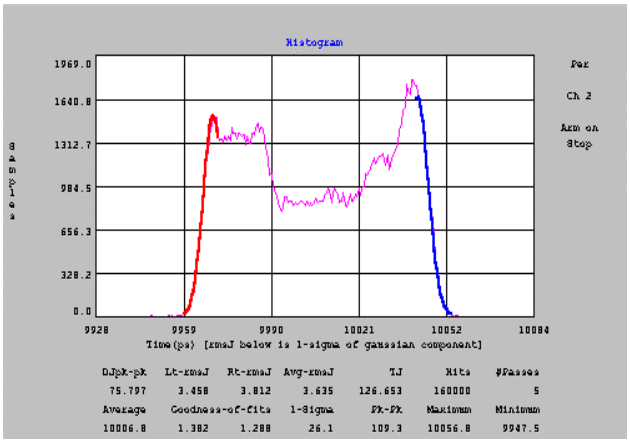

Figure 4 - Histogram tail search and fitting algorithm application for clock jitter, using Wavecrest's DTS system as the measurement instrument.

Using the DJ and RJ values, clock performance can be predicted with the bit error rate error (BER) curve that is calculated through the measured total histogram and RJ number. Please see reference [3] for details on the how to calculate the BER curve. Figure 5 shows the BER curve (also called bathtub curve). Thick lines indicate the actual measured BER and thin lines indicate the extrapolated BER based on a RJ Gaussian PDF. BER curve is an important overall performance indicator for time critical ICs and systems. In serial data communication, total jitter is normally specified at an error probability level of 10^{-12}. In Figure 5, the operational margin is 9873.4ps, and the total jitter is 126.6ps at 10^{-12} probability level.

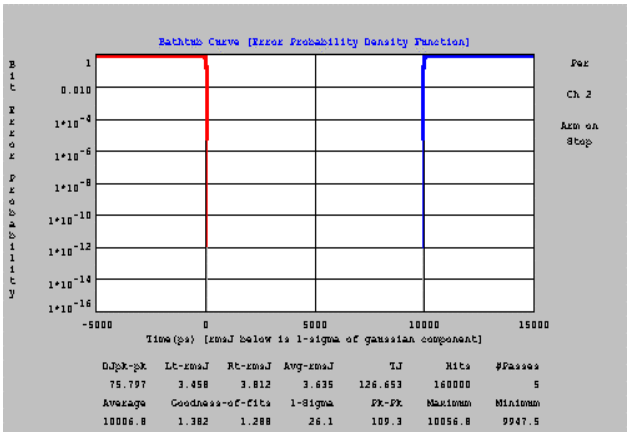

Figure 5 - BER curves for the same clock signal.

A correlation study has been carried out comparing the RJ and DJ values with those obtained by using the Blackman-Tukey method. In most cases, the difference was less than 5%. However, tail fitting methodology does not require the time span of the jitter record. Consequently, tail-fitting algorithms have a wide application in the field of jitter analysis.

Summary and Conclusions

WAVECREST Corporation has developed a general-purpose, automated search and nonlinear fitting algorithm with special emphasis on deterministic jitter and random jitter separation. Under significant statistical fluctuation, this algorithm separates DJ and RJ accurately and repeatedly. This algorithm does not require any user intervention and applies in both laboratory and production applications. This algorithm can apply to either a single histogram, or a series jitter histogram (for deterministic and random jitter spectrum analysis).

These algorithms can also be useful in other general-purpose signal analysis applications, too. For example, this method can be used to analyze phase noise spectrum and determine what kind of noise processes are involved in a specific device such as clock PLL or clock recovery PLL. Other examples include: DJ and RJ separation for eye-histogram and bounded uncorrelated jitter (BUJ) separation that can be caused by cross talk.

WAVECREST Corporation has also simulated the effect of sampling statistics error using the Monte Carlo method, for a single Gaussian distribution. This is a very important issue since the sampling statistics used to represent the underlining population statistics can only be measured. Regardless of what kind of analysis tool one is used, if the sampling statistics are far from the population statistics, the true value for the population statistics will be inaccurate. This is true for any kind of measurement. The goodness of the sampling statistics is proportional to the total number of measurements used to compose the histogram. As the total number of measurements increases, the better the sampling statistics. If the histogram measurement and the DJ and RJ value deduction process, for a given total number of measurement per histogram, is repeated for a given number of times, the DJ and RJ parameters deduced will compose distributions that are very close to Gaussian. The standard deviation for the DJ and RJ distributions obtained in this manner is inversely proportional to the total number of measurements in each histogram composition. *WAVECREST*'s simulation has shown a 1σ error of 4.5% for DJ and 17.2% error for RJ, given 10,000 measurements per histogram and repeating the simulation 100 times. Of course, for different histograms or for different measurement totals, these numbers can change. The point is, sampling statistics are very important for guaranteeing the accuracy of DJ and RJ values. A minimum number of measurements are needed for quality sampling statistics. However, that minimum number can vary with different applications and accuracy specifications.

References

[1] Y. Takasaki, “Digital transmission design and jitter analysis,” Artech House, Inc., 1991

[2] P. R. Trischitta, E. L. Varma, “Jitter in digital transmission systems,” Artech House, Inc. 1992.

[3] National committee for information technology standardization (NCITS), working draft for “Fiber channel – methodologies for jitter specification,” Rev 6, 1998.

[4] J. Wilstrup, “A method of serial data jitter analysis using one-shot time interval measurements,” ITC Proceeding, 1998.

[5] P. R. Bevington, D. K. Robinson, “Data reduction and error analysis for the physical sciences,” McGraw-Hill, Inc, 1992.

[6] Knuth, D. E. “Semi-numerical Algorithm,” 2nd edition, Addison-Wesley, 1981.

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